Limit Analysis for the Calculation of Natural Stone Vault Bridges

F. PURTAK[1]

ABSTRACT:

Primary concern while analysing the bearing capacity of Stone Vault Bridges is to estimate as realistic as possible the capacity of mostly historical valuable and robust structures and to support their conservation by cost-efficient restorations. Using the example of a natural stone bridge of ashlar masonry a new method has been developed. This method is based on bearing load curves. It is derived with the so called "Decoupled Discontinuum Model".

On the basis of existing families of capacity curves the cross-section capacity of common natural stone masonry can be read off in consideration of the real geometry and material parameters. Bearing loads are determined at a Micromodel with non-linear material parameters. The Mohr-Coulomb failure criterion is used for stones. Under assumption of plastic characteristics of the mortar the Drucker–Prager yield criterion is used. The stress of the vault construction is displayed by calculation of the bearing surface with the stress path at the critical cross-section. The intersection point of the stress path and the design capacity curve yields to the utilisation factor of the construction (in comparison to the needed partial safety factor).

The calculated collapse loads using the "Decoupled Discontinuum Model" correspond well with experimental tests at the structures in full scale.

Keywords: *Discontinuum Model, bearing surface, cross-section bearing capacity, capacity curves, partial safety factor, load bearing test*

NOTATION

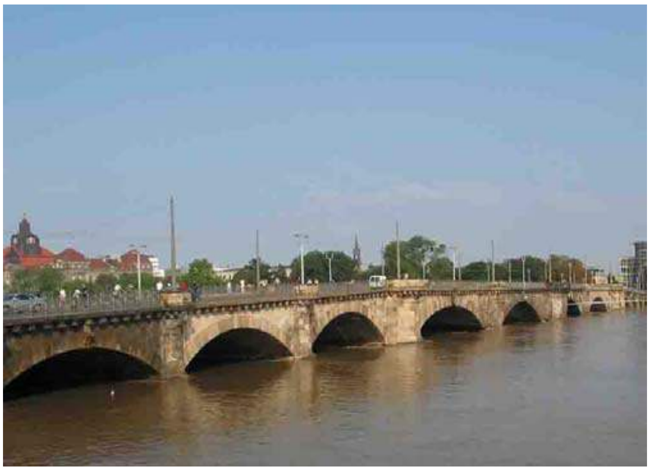
e, m	eccentricity of the normal force, specific eccentricity
d	thickness of the vault
$\beta_{Z,St}$, $\beta_{D,St}$	tensile strength, compression strength of stone

1 INTRODUCTION

Vault bridges made of natural stone masonry (Figure 1) rank among the oldest structures still in full function within road and railways. It is estimated that there exist approximately 70.000 railway bridges in Europe made of stone masonry [1]. There are hardly any information about the number of road bridges even with longer-distance span width, as road departments are locally organised.

[1] Dr.-Ing., Trag Werk Ingenieure, Döking + Purtak Partnerschaft, 01309 Dresden, Germany, FP@tragwerk-ingenieure.de

In course of the revision of these structures, evaluations regarding load-carrying capacity and serviceability are demanded on the basis of today's valid loads and standards. Current tools for structural calculations are meanwhile quite versatile. They enable civil engineers to regard geometrical as well as physical non-linear building parameters.



a) Albertbrücke in Dresden at high water mark

b) Road bridge in Zurich

c) Road bridge in Linz

d) Bridge over the Moldau in Prague

Figure 1. Natural Stone Vault Bridges

For the analysis of the bearing capacity available standards unfortunately allow only a general assessment of resisting forces. In this respect there is a noticeable deficiency between the options of the mechanical modelling and the available proofs of safety.

The following shown bearing load method should enable a realistic and clear calculation with the aid of the "Decoupled Discontinuum Model". It presumes most possible exact knowledge of the essential building parameters (geometry, strength values) and a bearing condition analysis. One focal point is put on the calculation of the masonry's capacity for real occurring geometry and strength values. These values have been edited in the form of universal valid bearing load curves.

2 DETERMINATION OF STRESSES ON BEARING STRUCTURES

2.1. Actions on structures

The characteristic values of the standard load models according to Eurocode 1 [2] respectively the DIN-Fachbericht 101 [3] are interpreted as 98 %-fractile. For instance the wheel load of 120kN for road bridges corresponds to the theoretical averaged 50 years value. Vibration coefficients are already included in the load definition for road bridges. However they have to be considered additionally for railway bridges. Structural characteristics for live loads resulting from road or railway traffic can be expressed well by generalized extreme value distributions. These characteristics are combined with the prospective safety factor according to DIN 1055-100 [4] in the probabilistic calculation for determining the partial safety factor (see Table 1).

2.2. Calculation of Vault Structures as Discontinuum

The calculation of the bearing capacity of natural stone masonry with the "Decoupled Discontinuum Model" is based on the consideration of stones and joints. For vault bridges the construction is modelled by volume elements and the joints are simulated as contact elements which allow only pressure and Coulomb friction forces but no tension forces. The joints between the ashlar stones have been kept open in transverse direction of the bridge to meet the real situation. They are therefore not linked within the grid (Figure 2).

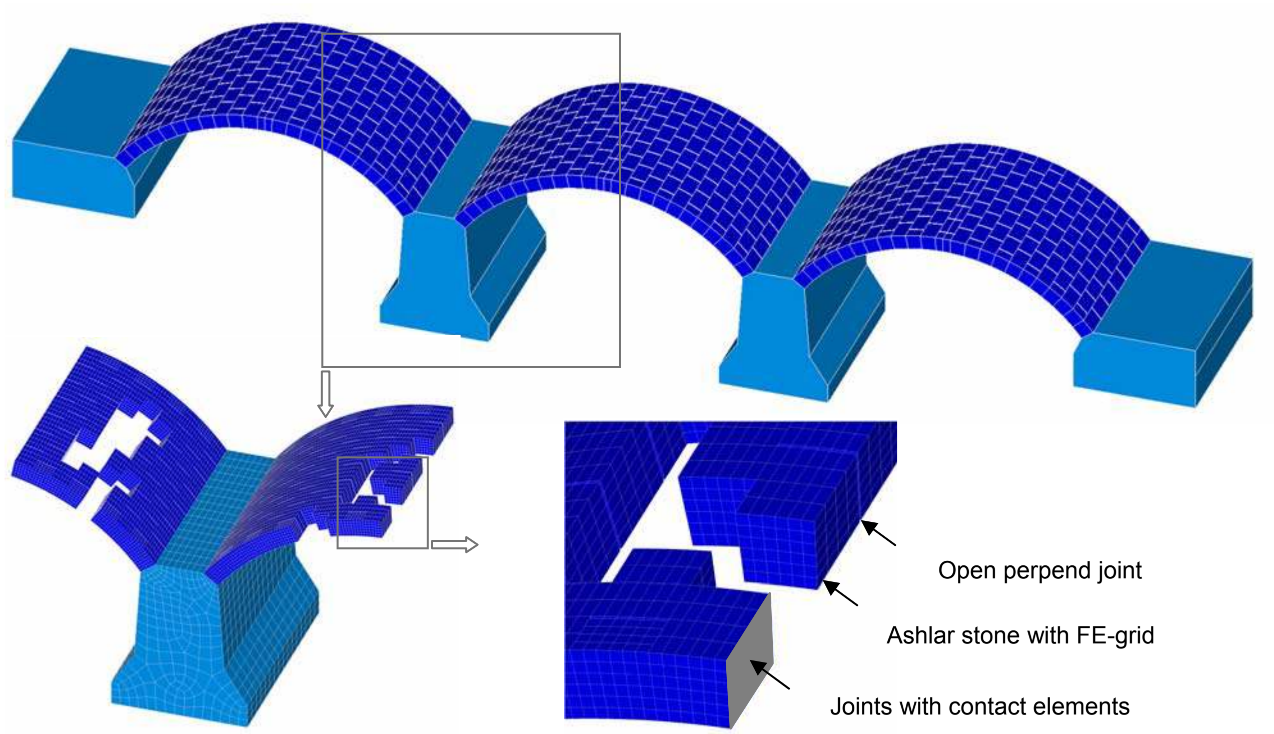


Figure 2. Vault bridge with ashlar stones and joints linked by contact elements; Joints are open in transverse direction of the bridge; backfill not shown

Consequently stresses can only be transmitted in transverse direction of the bridge by friction in the overpressurised joint. Under load the "bearing surface" automatically evolves „gaping joints". This bearing surface corresponds at self-weight with the line of pressure of an 1m-strip. When there are imposed loads which are unsymetrical in transverse direction of the bridge, the bearing surface will adjust itself according to the load situation.

The Mohr-Coulomb failure criterion is used for stones. Under assumption of plastic characteristics of the mortar the Drucker–Prager yield criterion is used. The collapse load of the vault can be calculated at the entire system for the hypotheses stone collapse and the appearance of an additional hinge, which leads to a kinematic system. The comparison of the collapse and the standard load shows the corresponding structural safety. Every load position and load combination requires a particular calculation, as the law of superposition can't be applied due to nonlinearities.

At the following shown "Decoupled Discontinuum Model" the load and capacity of vault bridges is analysed at two separate FE-models (Figure 3):

- The FE-model of the bridge contains the ashlar stones and the potential of gaping joints.
- The capacity of the masonry cross section is determined at a separate Micromodel with stones and mortar considering the physical non-linear characteristics [6]. Using the FE-program ANSYS [7] extensive families of capacity curves have been supplied for common used natural stone masonry. They are results of the research project [5]. In consideration of the partial safety factor they produce the bearing capacity curves for the statical proof.

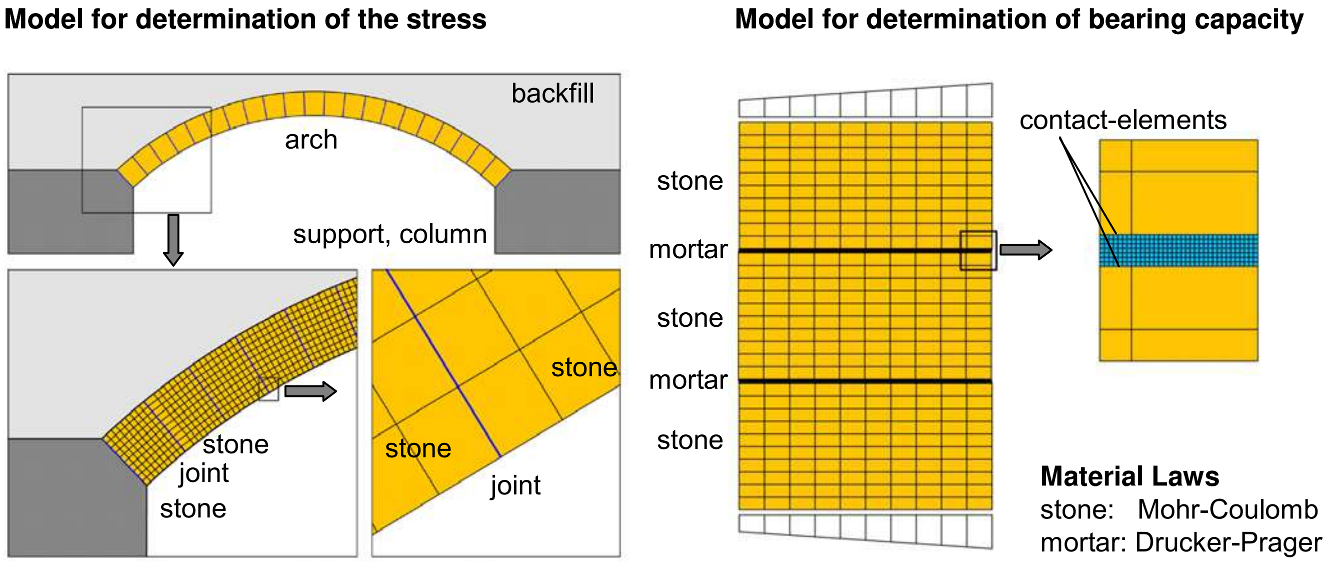
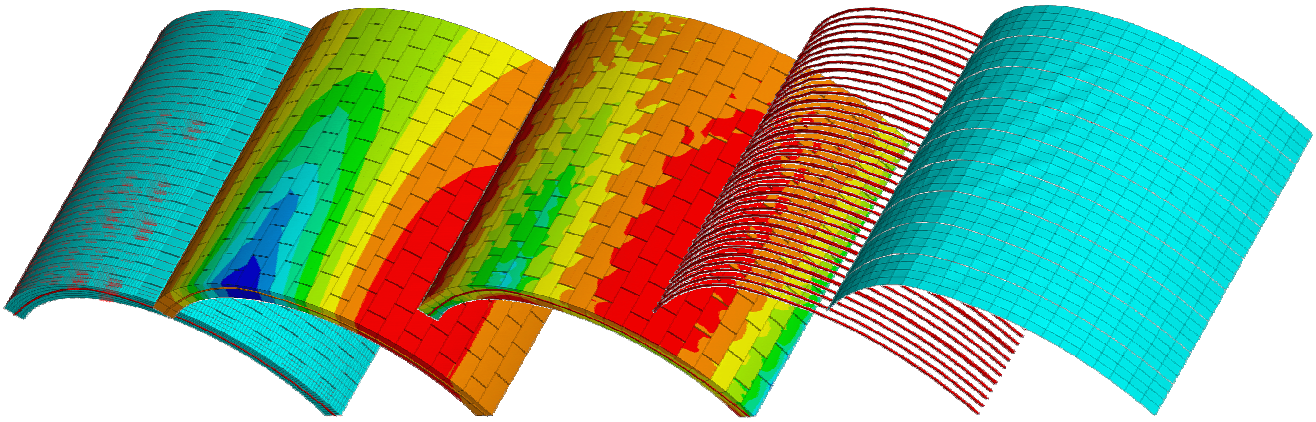


Figure 3. Principle of the "Decoupled Discontinuum Model"

Bearing surfaces from different load positions of the moving load can be determined with the FE-model. Then they can be compared with the supplied design capacity curves.

For the determination and graphical representation of the bearing surface it is necessary to do integration over all masonry joints over the distribution of the compressive stress in direction of the vault's thickness and to determine the centre of gravity of each distribution. Figure 4 shows for a part of the vault an FE-model under load of load model LM1. The loads are entered at the FE-node at the vaults surface (small arrows).

The deformation and the distribution of the principle compressive stress show the unsymmetrical load position. The lines of pressure are calculated by a narrow grid of longitudinal sections through the vault and are then shown as bearing surfaces. It is possible to verify the bearing surface with the aid of a linear elastic system, as long as there are no gaping joints (criterion $m = 6 \cdot e/d \leq 1{,}0$).



a) FE Model under load b) deformation c) principle compressive stress d) pressure lines e) bearing surface

Figure 4. Determination of the bearing surface from the principle compressive stress in consideration of gaping joints

3 BEARING CAPACITY OF MASONRY WITH THE AID OF CAPACITY CURVES

FE calculations based on the “Decoupled Discontinuum Model“ require knowledge about the bearing capacity of masonry in consideration of the eccentricity of loads. For certain types of stone geometries and joint thicknesses it can be derived by methodical determined capacity curves. The usage of bearing capacity curves has two essential advantages: the compressive strength at centric load application is base of the calculation and the bearing capacity is given for every eccentricity of loads of the real masonry.

These kind of bearing capacity curves describe the bearing capacity for the eccentricity of loads $m = 6 \cdot e/d$ from $m = 0$ (centric load) till $m = 3$ (edge load). For the description of the material behaviour of the mortar the elastic-ideal plastic Drucker–Prager yield criterion is used. The Mohr-Coulomb failure criterion is used for the natural stone.

The Finite Elemente Model consists of stones and mortar joints, which are linked with 4-node elements for the plane strain state. Consequently deformations normal to the panel plane have been eliminated, but the triaxial state of stress has been considered. Compared to the stone and its linear elastic material behaviour, the joint has to be cross-linked more densely due to the nonlinear material behaviour of the mortar (plastic deformable zones need an sufficient density of integration points).

The bearing capacity is affected significantly by the material parameters of the stones: compression and tensile strength as well as the geometry parameters of the stones: height and thickness. Therefore it is obvious to create a tridimensional family of curves (Figure 5). They contain the bearing capacity related to the compression and tensile strength of the stones as well as to the eccentricity of loads for a certain stone thickness. Intermediate values can be determined by interpolation.

To verify the quality of the results of the FE calculations, experiments [8] at Three-Stone-Bodies have been conducted (lime-sand brick: B/L/H = 62,3/34/30 cm; joint thickness t = 1,5 cm). The results of the FE calculations are on the safe side.

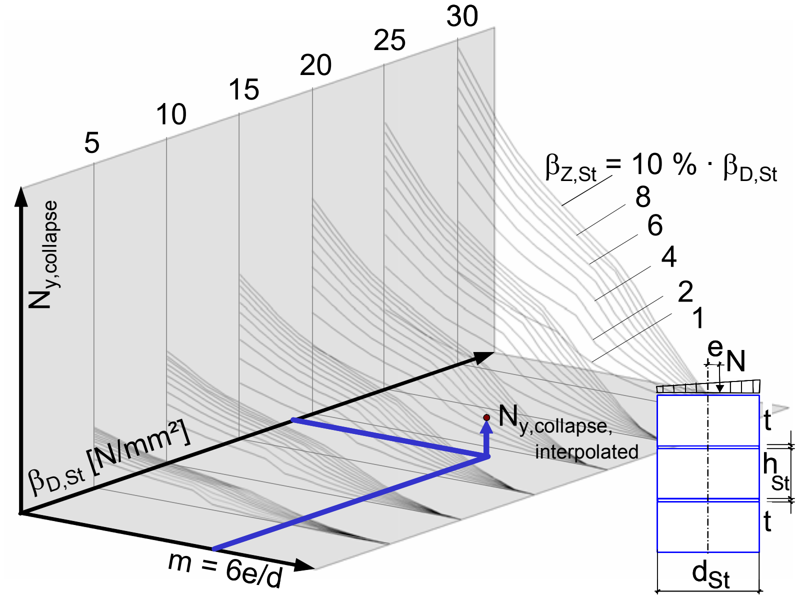


Figure 5. Family of capacity curves in relation to the material and geometry parameter

4 SAFETY FACTORS

For the proof of structural safety with the aid of the “Decoupled Discontinuum Model“ there have been done numerous probabilistic calculations with dispersive actions and dispersive material parameters [5]. Table 1 shows the partial safety factors. The characteristic values of the action respectively the material parameters have to be multiplied with or divided by these factors. The determined partial safety factors are based on the stress resultant oriented process of proof:

1. Calculation of the stress (design stress resultants);
2. Determination of the decisive bearing capacity curve with partial safety factors (design capacity curve);
3. Comparison stress ≤ capacity.

Table 1. partial safety factors for the analysis with bearing capacity curves [5]

Action	γ_f	Comment
Road traffic (characteristic value)	1,30 1,20	Calculation class for existing bridges according to DIN 1072; Special vehicles after special specifications
Railway traffic (characteristic value)	1,30 1,20 1,10	UIC 71 (after Ril 805); Special vehicles, range class after special specifications; For all additional loads (after Ril 805)

Action	γ_f	Comment
self-weight (mean value)	1,10 0,90	at unfavourable effects for the structural safety at favourable effects for the structural safety
temperature (mean value)	1,00	
Support Subsidence (mean value)	1,00	according to the geotechnical report; not necessary at arches free of failure with long life time without increasing of subsidence-related loads
structure parameters *)	**γ_m**	**Comment**
structure geometry (mean value)	1,00	from as-built documentations or measurements on site
Elastic modulus of the materials (mean value)	0,90 ... 1,10	calculation optionally for the upper and lower limit; use for mean value 1,0
geometry of stone and joint (mean values)	1,00	always to be checked; estimate joint thickness conservatively
compression strength of stone**)	1,30	related to the 5 %-fractile (characteristic value); limit of validity for coefficient of variation: $v = \sigma / m < 30$ %
tensile strength of stone **)	1,80	related to the 5 %-fractile (characteristic value); limit of validity for coefficient of variation: $v = \sigma / m < 30$ %
compression strength of mortar (mean value)	1,00	uniaxial compression strength

*) The term „structure parameters“ has been chosen deliberately, as some parameter can be classified as “resistance” and as “action” after the probalistic definition.

**) For sandstone it is recommended to prove according to material testing, that the tensile strength is at least 5% of the compression strength (mean value).

5 VERIFICATION USING THE EXAMPLE OF THE “DECOUPLED DISCONTINUUM MODEL“

At an example structure the described “Decoupled Discontinuum Model“ will be used for the statical verification. The load combinations which have be examined, consist of permanent loads and of the load model LM1, which has been put on the bridge as moving load.

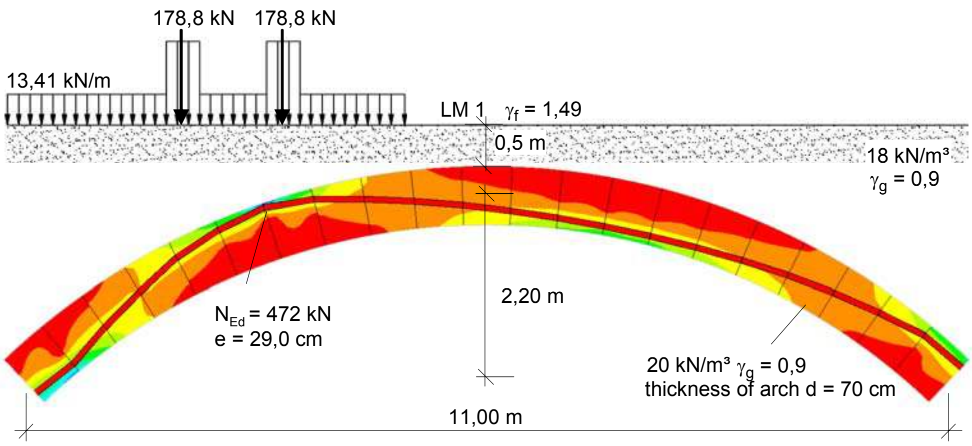


Figure 6. Calculation with “Decoupled Discontinuum Model“ for design loads; longitudinal section through vault and bearing surface in the most stressed region

The loading is increased gradually for the chosen load position and the corresponding bearing surface is determined (characterised by the axial force N and the specific eccentricity m). At the relevant section of the vault the intersection point of the calculated stress and the bearing capacity curve is determined (Figure 7). The bearing capacity curve results from the 5 %-fractile values of the tensile and compression strength of the stones, which have been divided by the partial safety factors. The bearable load must be at least as much as the characteristic load of load model LM1 multiplied by the partial safety factor γ_f.

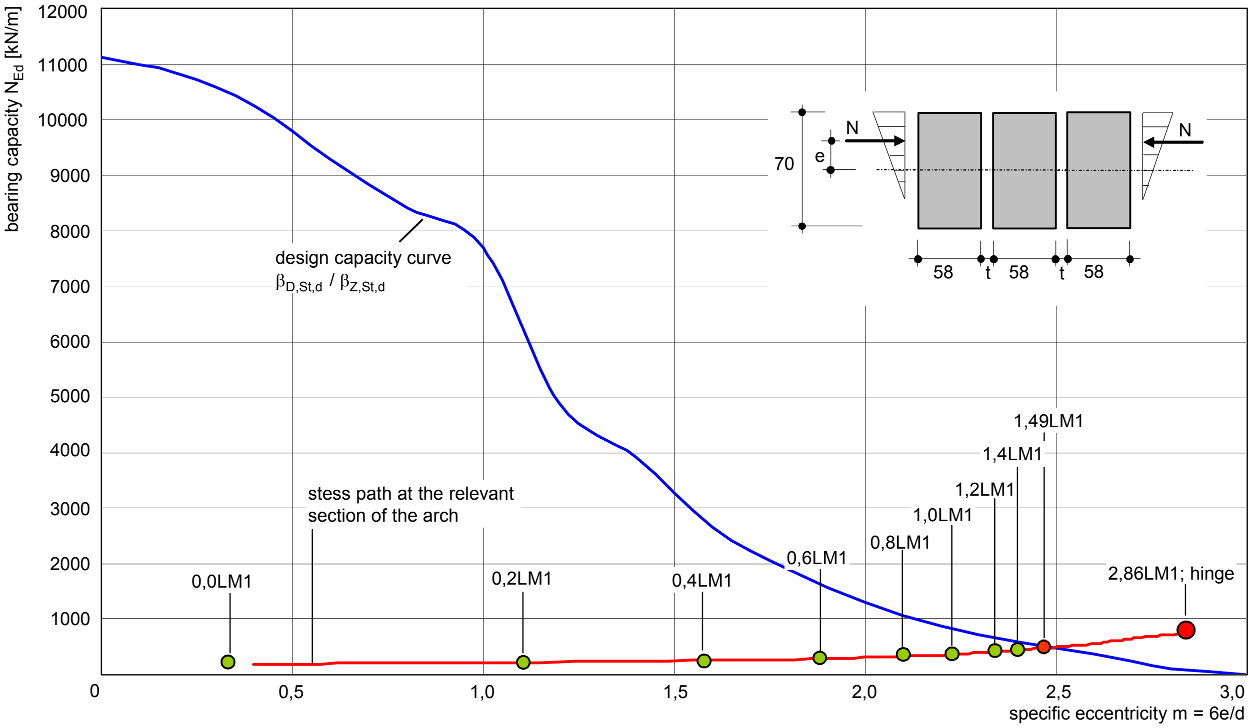


Figure 7. Design capacity curve and stress path of the relevant section

The intersection point of the stress path of the most stressed section in the vault and of the design capacity curve shows the design bearing capacity of the bridge. The bearing capacity is 1,49 times bigger than the characteristic load value.

The characteristic load of load model LM1 can be increased further by 49 percent until the design bearing capacity is reached. The structural safety with the chosen partial safety factor is then γ_f = 1,3:

Q_{max}/ Q_{LM1} = 1,49/ 1,3 = 1,15 > 1

The reserve of bearing capacity is 15 percent. The maximum allowable eccentricity of m = 6 · e/d = 2,5 according to UIC-Kodex 778-3 [9], which is recommended to be used here, has been met (m = 2,49). According to DIN 1053-100 [4] the statical proof failed for the shown example, whereas the proof after UIC-codex is wide on the safe side. These two used methods show extremely different results [11].

Figure 8 shows the calculation of the collapse loads related to the tensile strength of the stone at the chosen compression strength of 25 N/mm². They are expressed as the multiple γ_Q of the load model LM1 for different widths of the bridge. The load model is put on the particular traffic lane for the width of 3, 6 till 12 meter. In comparison an arch with an 1 m strip has been evaluated. The increasment of the collapse load between a width of 6 and 7 meter can be seen clearly, because only two traffic lanes with two times 3,0m are under load according to the valid regulations. Wider bridges don’t yield to more increasement of the load, as the transverse distribution of loads decreases. The curve of the appearance of hinges represents the upper limit, to which the curves converge at increasing compression strenth of the stone.

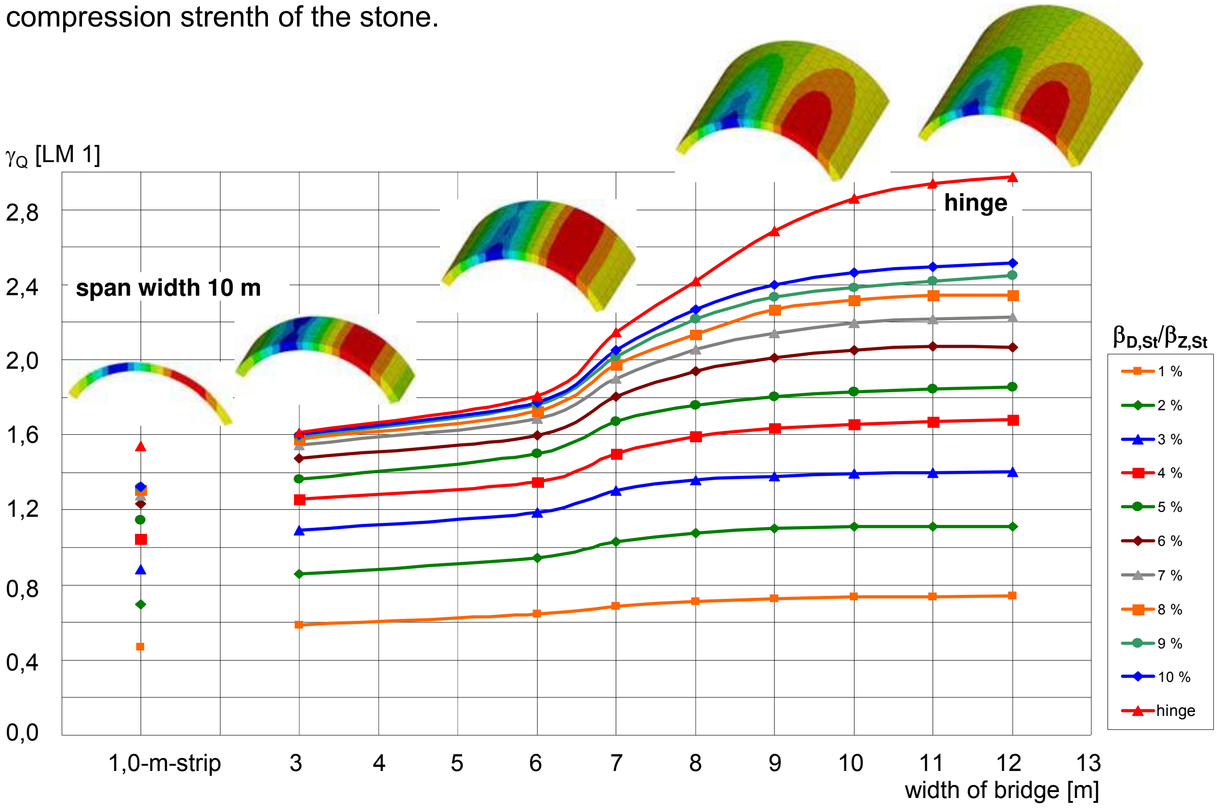


Figure 8. Collapse situation for the multiple γ_Q of the load model LM1 $\beta_{D,St}$ = 25 N/mm²

6 LARGE SCALE EXPERIMENTS

In the context of the research project [5], collapse load tests [10] have been performed complementary at two natural stone arches of 5m span. According to the load model LM1 the load had been entered by tension rods in the forth point of the span width. The position of the calculated bearing surface when the collapse load is reached as well as the position of the breaking point according to the FE calculation correspond well with the experimental test (Figure 9).

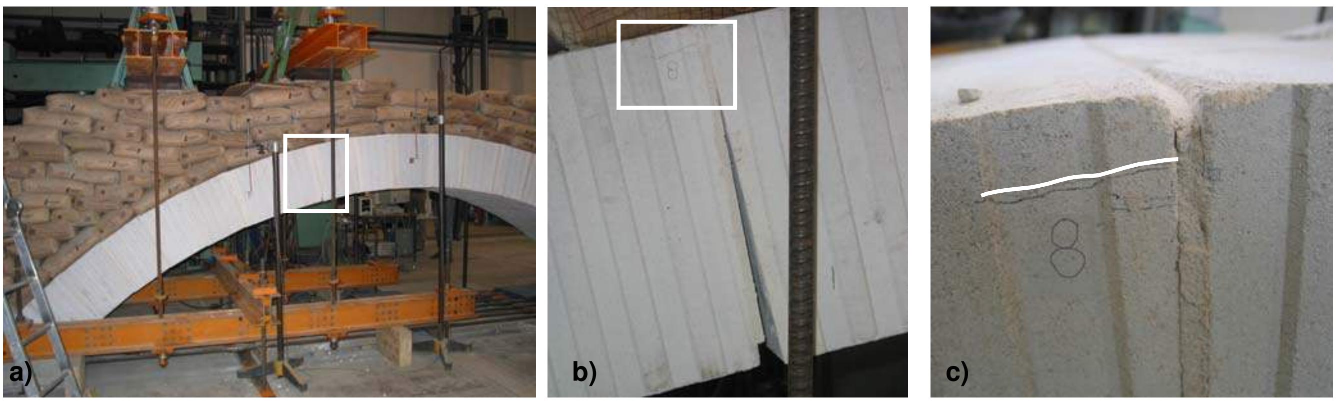


Figure 9. Crack orthogonal to the joint
a) Region of the crack b) Crack and gaping joint c) Detail of the crack

7 PERSPECTIVES

In spite of the current computer technology, the calculation with the developed Discontinuum Model is still very time-consuming. Therefore the statical proof at practical applications is made at a thought cut 1,0m model of an arch [5]. However the reserves of bearing capacity due to transverse distribution can't be considered. With the aid of parameter tests the vault model is compared to the arch model. It is to be examined at which load difference the utilisation factor is equal. Consequently the spatial reserves of bearing capacity of the vault model can be used by a load reduction factor when calculated at the simplified arch model. The reserves of bearing capacity which otherwise have been neglected, can be activated.

ACKNOWLEDGEMENTS

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